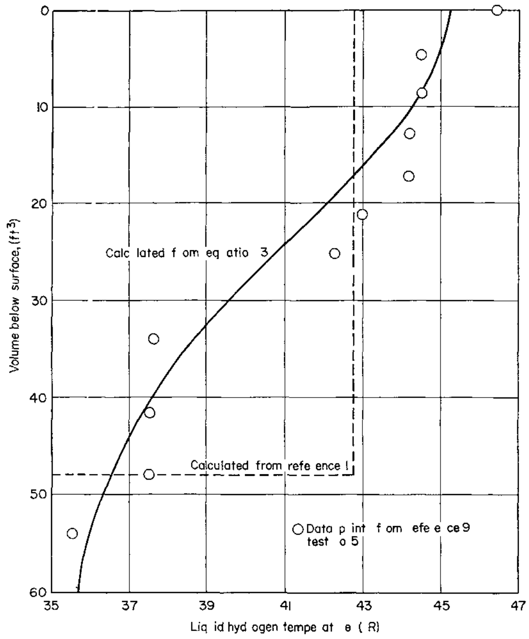

Fig 4 Stratification in a liquid hydrogen tank

evaporated (or condensed) propellant can be found using a method outline by Knuth [7] For smooth-wall tanks, there is insignificant mixing between the boundary layer and the bulk liquid since the convective layer at the wall acts as a channel directing flow to the surface Thus, for most cases with smooth-wall tanks, the assumption that I is equal to one is satisfactory

If the tank contains horizontal structural members or anti-slosh baffles attached to the wall, the boundary layer growing at the wall will be diverted into the bulk liquid For this nonsmooth wall configuration, the heat absorbed by the bulk liquid cannot be calculated with any degree of accuracy At present there are no published theories or experimental data on the effect of a baffle on a free convection stream One approximation that can be used is to assume that the flow around the edge of a baffle is flow from an infinite line source and use the data obtained by Rouse[8] on air to calculate the spreading effect of the baffle

For the case of an ullage pressurized with vapor derived from the liquid phase, the surface temperature is for all practical purposes the saturation temperature at the existing ullage pressure For other cases the surface temperature will be between the bulk temperature and the saturation temperature If the surface temperature cannot be calculated, little error is usually introduced by assuming that the surface temperature is saturated at the ullage pressure

Figure 3 compares the calculated and measured temperature profiles in a liquid-nitrogen tank subjected to simulated aerodynamic heating It can be seen that good agreement is obtained even though the ullage was pressurized with helium gas and the surface temperature was less than the saturated temperature at the given ullage pressure It was assumed that all the heat input was absorbed by the stratified layer ($I = 1$) Comparison with data obtained for LH_2 also shows good agreement with calculated results (Fig 4) Also plotted on these figures are the calculated results using the method developed in Ref 1

The effect of vehicle slosh, even at the resonant frequency, was found to be minor [9] In addition, data indicate that the temperature profile did not change appreciably during tank drainage, i e , converting the temperature profile to an outlet temperature as a function of time showed good agreement with the previously measured profile in the tank

It appears that the suggested temperature profile [Eq (1)] using the calculated constant [Eq (4)] yields results that agree satisfactorily with test data obtained with various liquids For tanks with structural members or anti-slosh baffles that interfere with the boundary layer convective flow, the calculation of the amount of stratification is much more difficult However, the use of baffles suggests that the degree of stratification can be controlled by proper design of horizontal structural members which will introduce mixing of the boundary layer with the bulk liquid Further experimentation is needed in the area of nonsmooth walls to evaluate the effect such members have on thermal stratification

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Determination of the Mass of Gas in a Rapidly Discharging Vessel

R C PROGELHOF*
Lehigh University, Bethlehem, Pa

Nomenclature

a = speed of sound
A = cross-sectional area
k = ratio of specific heats
L = length of vessel
m = mass
P = pressure
t = time
u = flow velocity
V = volume
x = distance along axis of vessel
ρ = density
ϕ = A_e/A_t

Subscripts

e = conditions at minimum cross sectional area of constriction
f = conditions at minimum cross sectional area of flow stream
i = conditions just ahead of constriction
0 = initial conditions in the vessel
s = surroundings
t = vessel

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* Assistant Professor, Department of Mechanical Engineering Member AIAA

Introduction

MANY of the discrepancies between experimental results and the calculations based on quasi-steady classical thermodynamics for the rapid discharge of a gas from a cylindrical vessel can be explained by the application of the wave theory of gas dynamics As early as 1940, Giffen,[1] using a numerical technique, showed that the pressure variation at a point in the vessel varied in a step-wise fashion instead of a smooth continuous curve as predicted by the classical approach Experiments on two-stroke engines conducted by Weaving[8] indicated that the discharge coefficients of the ports during nonsteady flows based on quasi-steady calculations were greater than discharge coefficients under steady flow calculation Kestin and Glass,[4] basing their analysis on the method of characteristics for nonsteady gas flow,[2 3 7] explained the discrepancies as mainly owing to the omission of the considerable velocity of approach The theoretical analysis of Kestin and Glass was limited to a semi-infinite vessel under sonic discharge, whereas the case of the finite vessel was only mentioned

The approach presented in this note to determine the mass of gas remaining in a rapidly discharging vessel of finite length was based on the graphical solution of the method of characteristics The boundary conditions used in the state plane, thus fixing the properties of the fluid entering the constriction and the strength of the rarefaction waves within the vessel, were constructed from previous results based on experimental pressure measurements within the vessel [5 6]

Theory

The mass inside the vessel at time t is equal to

$$m_t = m_0 - \int_0^t \rho_i u_i A_t dt \qquad (1)$$

$$\frac{m_t}{m_0} = 1 - \int_0^{a_0t/L} \left(\frac{\rho_i}{\rho_0}\frac{u_i}{a_0}\right) d\left(\frac{a_0t}{L}\right) \qquad (2)$$

This integral cannot be integrated directly but must be obtained by a graphical integration The values of $\rho_i u_i/\rho_0 a_0$ vs a_0t/L are obtained from the state and physical planes of the graphical solution

To illustrate the calculation technique and to compare the results with those predicted by the quasi-steady theory, two examples for the same area ratios, $\phi = 0\ 50$, and initial conditions, $P/P_0 = 0\ 0824$, are worked out, first for a nozzle and second for an orifice The state and physical planes are only presented for the first example, the discharge through a nozzle

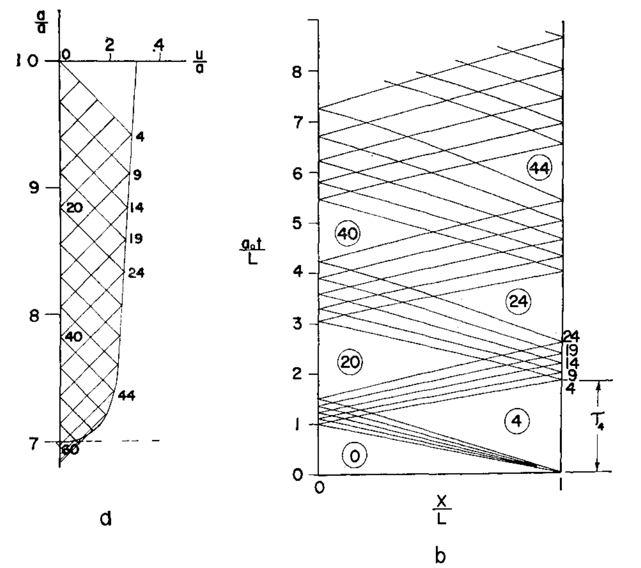

Fig 1 **Graphical solution for the discharge through a nozzle under the following boundary conditions: $A/A_t = 0\ 50$, $P/P_0 = 0\ 0824$; a) state plane, b) physical plane**

The boundary conditions in the state plane (Figs 1a and 1b) are constructed [6] The variation of the flow properties u and a across the initial rarefaction wave entering the vessel will lie on a Γ^+ characteristic passing through the origin of the state plane, point 0, and the intersection of the ϕ = constant boundary condition, point 4 Point 4 also represents the flow properties behind the initial rarefaction wave The flow will remain steady and uniform into the constriction area ④ of the physical plane, until the initial rarefaction wave is reflected from the sealed end of the vessel and reaches the mouth of the constriction, point 4 on the physical plane The variation of flow properties across the wave reflected from the closed end will lie on a Γ^- characteristic passing through point 4 and the intersection of the a/a_0 axis which satisfies the second boundary condition of zero flow velocity at the sealed end, point 20 Thus, the time duration for the steady and uniform flow, ④, into the constriction is τ_4 The density of the exiting fluid is calculated from the isentropic relationship:

$$\rho/\rho_0 = (a/a_0)^{2/(k-1)} \qquad (3)$$

The dimensionless factor, $\rho_i u_i/\rho_0 a_0$, can be plotted vs a_0t/L (Fig 2) for the first nondimensional time interval τ_4 The flow properties of the fluid entering the constriction during the reflection of the initial rarefaction from the mouth of the constriction (points 9, 14, and 19 of Fig 1b) change continuously until the wave is completely reflected and steady and uniform flow again persists, ㉔ The dimensionless factors are calculated in the same manner and the diagram completed (Fig 2) The graphical solution for the discharge through an orifice, based on the boundary conditions of Ref 5, is similarly constructed and the dimensionless factor, $\rho_i u_i/\rho_0 a_0$, plotted in Fig 2 This figure is then graphically integrated, and the nondimensional mass m/m_0 vs time a_0t/L relationship is obtained (Fig 3) The slope of the mass-time curve for the discharge through a nozzle, the mass flow rate out of the vessel, is plotted in Fig 4

Quasi-Steady Theory

The quasi-steady theory of a discharge of a gas from a vessel, which has been developed by Giffen[1] and extended by Weaving,[8] is well known Under the assumption of quasi-study conditions inside the vessel and sonic discharge one can deduce the following relationships for air to determine the pressure and the rate of mass discharging as a function of time:

$$\frac{\phi a_0 t}{L} = 8\ 64\left[\frac{1}{(P_t/P_0)^{1/7}} - 1\right] \qquad (4)$$

$$\frac{L(dm/dt)}{a_0 m_0 \phi} = 0\ 5788\left(\frac{P_t}{P_0}\right)^{6/7} \qquad (5)$$

Similarly, for subsonic discharge,

$$\frac{t}{(L/a_0\phi)\,(P_t/P_e)^{1/7}} = 0\ 2795\Bigg[3\ 946 - \Bigg\{\Bigg[\left(\frac{P_t}{P}\right)^{2/7} - 1\Bigg]^{1/2} \times \Bigg[2\left(\frac{P_t}{P}\right)^{2/7} + 3\Bigg]\left(\frac{P_t}{P}\right)^{1/7} + 3\ln\Bigg(\Bigg[\left(\frac{P_t}{P}\right)^{2/7} - 1\Bigg]^{1/2} + \left(\frac{P_t}{P}\right)^{1/7}\Bigg)\Bigg\}\Bigg] \qquad (6)$$

$$\frac{L\ (dm/dt)}{a_0 m_0 \phi} = 2\ 236\left(\frac{P_e}{P_0}\right)^{6/7}\left[\left(\frac{P_t}{P}\right)^{2/7} - 1\right]^{1/2} \qquad (7)$$

Equation (6), the integrated form of the subsonic pressure-time relationship, was obtained from Ref 9 The mass re-

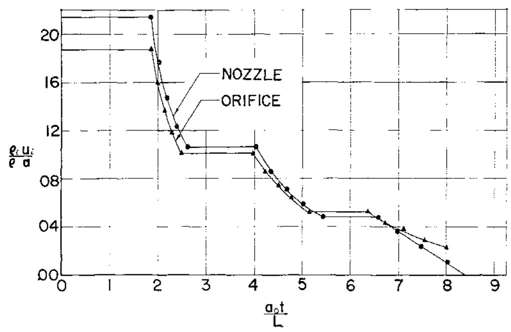

Fig 2 The dimensionless factor, $\rho_i u_i/\rho_0 a_0$, vs nondimensional time

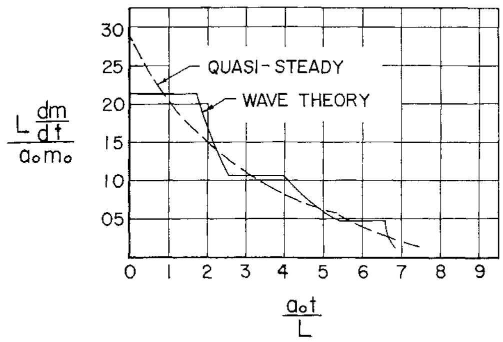

Fig 4 A comparison of the quasi-steady and wave theory mass discharge rates for the discharge through a nozzle

maining in the vessel is obtained from the pressure relationship, Eqs (4) and (6), and the isentropic relationship:

$$m/m_0 = (P/P_0)^{1/k} \tag{8}$$

Introducing the velocity of approach factor obtained by Kestin and Glass[4] to the quasi-steady results for the discharge through a nozzle, and assuming a coefficient of discharge of one, the modified quasi-steady mass-time curve is plotted in Fig 3 For the sonic portion of the discharge process the velocity of approach factor was calculated continuously, whereas an average value was used for the subsonic process For the discharge process through an orifice, the quasi-steady approach which includes the velocity of approach factor is fairly difficult to apply due to the large variation of the discharge coefficient during the process However, for the sonic portion of the process, a fairly good representation of the process can be made by assuming a mean value for the discharge coefficient for the pressure range covered, 0 80 for the example under consideration (See Fig 7, Ref 5) The results are plotted in Fig 3

The quasi-steady discharge rate through the nozzle, slope of the mass-time curve (Fig 3), is plotted in Fig 4 for a comparison with the results based on wave theory

Discussion of Results and Conclusions

A comparison of the results (Fig 3) indicates that the mass-time relationship for the sonic discharge of a finite vessel through a nozzle can be closely approximated by the application of the velocity of approach correction factor to the quasi-steady results For the sonic discharge through an orifice the results depend on the estimate of the average discharge coefficient for the process In a subsonic discharge process the pressure in the vessel may fall appreciably below the pressure of the surrounding medium which the quasi-steady approach fails to predict In such cases there may be discrepancies between the quasi-steady and wave theory results In addition, the quasi-steady results predict a continuously changing mass flow rate, whereas wave theory predicts periods of constant mass discharge (Fig 4)

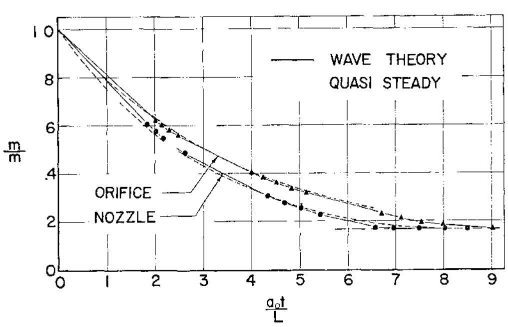

Fig 3 A comparison of the quasi-steady and wave theory results of mass vs time

In conclusion, the determination of the mass and mass discharge rate for a finite vessel can be calculated from the wave diagrams of the method of characteristics The modified quasi-steady technique of Kestin and Glass can be applied to the discharge of a finite vessel, and will closely approximate the mass based on wave theory for the sonic portion of the discharge process if the appropriate discharge coefficient is used

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Buckling Load of Bars with Variable Stiffness: A Simple Numerical Method

A V Srinivasan*
State University of Iowa, Iowa City, Iowa

THERE are occasions when the buckling load of a stepped bar, or a bar with varying moment of inertia along its length, is desired Classical methods are, no doubt, available, but all of them involve elaborate computations and/or theory The method presented here is simple and direct It is based on the finite differences technique in a way

The governing differential equation for a buckled bar with hinged ends is well known It is

$$EIY'' + PY = 0$$

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* Instructor